Response Time Analysis of Sporadic DAG Tasks under Partitioned Scheduling

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CISTER-TR-160502

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Abstract

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I. Introduction

The introduction of multi- and many-core architectures in the embedded domain has set up the basic environment for the deployment of modern applications, sharing both real-time and high-performance strict requirements. In order to exploit the high computation power provided by those architectures, these applications are typically implemented using parallel programming models (such as OpenMP). Such implementations, combined with the inherent complexity of the hardware, challenge the traditional timing analysis techniques that have been designed primarily in the embedded domain to analyze simple software codes, which were meant to run on simple and predictable hardware architectures.

Recently, the real-time community has started to actively study the timing behavior of parallel tasks under well-established scheduling algorithms. Several parallel task models and schedulability tests have been proposed for multiprocessor systems [1]–[3]. A common assumption among most of these works is that the task system is globally scheduled. Although global scheduling theoretically allows for an overall higher performance, it adds uncertainty and variability to the lower-level timing analysis, which then become overly pessimistic.

As noted in [4], on a multicore system there are strong inter-dependencies between timing and schedulability analysis, since the worst-case execution times are heavily dependent on the amount of cross-core interference generated on shared resources. This phenomenon is further exacerbated with parallel tasks due to the intense communication and concurrency between their sequential computational units. Thus, it is our belief that partitioned scheduling is the most promising approach to support parallel tasks in hard real-time systems.

Partitioned scheduling is a well-studied topic in real-time distributed systems. Different response time analyses, priority assignment techniques and mapping heuristics that allow for task parallelism have been proposed in the past years [5]–[8]. Although these works provide a strong understanding of the worst-case behavior of parallel tasks, they are inevitably less effective when applied to multicore systems due to the absence of the network component and local release jitters. Results concerning the schedulability of partitioned parallel tasks in multiprocessors are very limited. In [9], the authors presented a response time analysis for sporadic fork-join tasks with arbitrary deadlines under fixed-priority scheduling. Unfortunately, the paper was found flawed. Therefore, to the best of our knowledge only the results transposed from distributed systems provide an answer to the problem of scheduling partitioned parallel tasks atop multiprocessors.

This work. We consider a sporadic DAG model, where each sequential computational unit of a DAG task is assigned to a specific core. That is, multiple computational units can run in parallel over the multiprocessor platform but they are not allowed to migrate. We assume that such mapping is given. Under fixed-priority partitioned scheduling, we present a novel response time analysis for this task model. We observe that a partitioned DAG task can be modeled as a set of self-suspending tasks. Such self-suspending tasks depend on each other due to the precedence constraints defined in the DAG. To overcome this problem, we propose an algorithm to traverse a DAG and characterize the worst-case scheduling scenario. Moreover, we show how to transform existing response time analyses for sporadic self-suspending task in uniprocessors to analyze partitioned DAG tasks. In comparison to [6], the evaluation results show that our approach obtains substantial gains in terms of computed worst-case response time (WCRT).

II. Related Work

The problem of scheduling parallel tasks atop multiprocessor platforms has been receiving considerable attention from the real-time community. However the majority of the works have focused on global scheduling. Several parallel task models have been proposed to cope with the different forms of task parallelism generated by commonly used parallel programming models. In the fork-join model [9], [10], a task is represented as an interleaved sequence of sequential and parallel segments. Typically, each parallel segment contains the same number of subtasks, which in turn may not exceed the number of cores in the platform. The synchronous parallel model [1], [2], [11] extends the fork-join model by allowing successive parallel segments and an unconstrained number of subtasks within each segment. Nonetheless, synchronization is still assumed at every segment's boundary, so that no subtask is ready for execution unless all the subtasks of the previous segment have completed. A less restrictive parallel structure is supported by the DAG model [3], [12]–[14], where each task is instead characterized by a directed acyclic graph. Nodes represent subtasks and edges define precedence constraints between two nodes. In this sense, a subtask becomes ready for execution as soon as all its precedences constraints are satisfied. Recently, researchers have started addressing conditional parallel tasks [15]–[17] by taking into consideration the different flows of execution that a parallel task may experience.

Although partitioned scheduling of sequential tasks is a well-studied topic (see [18] for a comprehensive survey), results for parallel tasks are limited [5]–[9], [19]. In the context of distributed systems, Tindell and Clark [5] introduced an end-to-end schedulability analysis for a sequence of events called transactions, which was later refined by Palencia et al. [6]. Parallelism is expressed through these transactions. Enhancements to this analysis were then proposed in [7] by considering offsets and in [8] by considering precedence relations. EDF systems have also been addressed [19]. Axer et al. [9] derived a response time analysis for fork-join tasks under fixed-priority scheduling. To the best of our knowledge this is the only result available specifically for partitioned parallel tasks in a multicore setting. Unfortunately, the analysis in [9] was found flawed.

III. Model

We consider a set of n sporadic DAG tasks $\tau = \{\tau_1, \tau_2, \ldots, \tau_n\}$ to be scheduled under a fixed-priority partitioned scheme on a multiprocessor platform composed of m identical cores. We assume that τ_j has higher priority than τ_i if $j < i$. Each sporadic DAG task τ_i is characterized by a 3-tuple (G_i, D_i, T_i) with the following interpretation. Task τ_i is a recurrent process that releases a (potentially) infinite sequence of "jobs", with the first job released at any time during the system execution and subsequent jobs released at least T_i time units apart. Every job released by τ_i has to complete its execution within D_i time units from its release — we assume $D_i \leq T_i$ (constrained deadline system).

The structure of the jobs of τ_i is specified as a directed acyclic graph (DAG) $G_i = (V_i, E_i)$, where V_i is a set of n_i nodes and E_i is a set of directed edges connecting any two nodes. Each node $v_{i,\ell} \in V_i$ represents a computational unit (hereinafter referred to as "*subtask*") that must execute sequentially. A subtask is characterized by a worst-case execution time (WCET) $C_{i,\ell}$ and the core to which it is assigned $P_{i,\ell}$. That is, $v_{i,\ell} = (C_{i,\ell}, P_{i,\ell})$. In this work, we assume that each subtask can execute only on one core (subtask partitioning), and the subtask-to-core mapping to be given. Note that a mapping phase is essential to derive WCETs under such parallel settings due to the presence of shared resources and the magnitude of memory-alike transactions. Although the mapping of parallel tasks is an open problem, for schedulability purposes, it suffices to consider that the WCETs of the subtasks have been computed accordingly.

Each directed edge $(v_{i,a}, v_{i,b}) \in E_i$ denotes a direct precedence constraint between subtasks $v_{i,a}$ and $v_{i,b}$, meaning that subtask $v_{i,b}$ cannot start executing before subtask $v_{i,a}$ completes its execution. In this case, $v_{i,b}$ is called a "successor" of $v_{i,a}$, whereas $v_{i,a}$ is called a "predecessor" of $v_{i,b}$. A subtask is then said to be "ready-to-execute" (or simply "ready") if and only if all its predecessors have finished their execution. For any subtask $v_{i,\ell}$, its set of predecessors assigned to a particular core p is given by $pred(v_{i,\ell}, p)$. Analogously, $succ(v_{i,\ell}, p)$ returns the set of successors assigned to core p. Any two subtasks that are not predecessors/successors of each other, either directly or transitively[1], are called independent. Independent subtasks may execute in parallel whenever they are mapped to different cores. A node with no incoming or outgoing edges is referred to as "source" or "sink", respectively. Multiple source and sink nodes are allowed.

We now present additional notations and terminologies.

Definition 1 (Path). *For a given DAG task τ_i, a path $\lambda_{i,k} = (v_{i,a}, \ldots, v_{i,z})$ is a sequence of subtasks $v_{i,\ell} \in V_i$ where (1) $\forall v_{i,\ell} \in \lambda_{i,k} \setminus \{v_{i,z}\}$, $\exists! v_{i,s} \in \lambda_{i,k}$ such that $(v_{i,s}, v_{i,\ell}) \in E_i$, (2) $\forall v_{i,\ell} \in \lambda_{i,k} \setminus \{v_{i,a}\}$, $\exists! v_{i,r} \in \lambda_{i,k}$ such that $(v_{i,\ell}, v_{i,r}) \in E_i$, (3) $v_{i,a}$ is a source node, and (4) $v_{i,z}$ is a sink node.*

Informally, a path is a sequence of subtasks where there is a direct precedence constraint between any two adjacents subtasks. We denote by ℓ_i the number of different paths $\lambda_{i,k}$ that can be extracted from task τ_i, i.e. $k \in \{1, 2, \ldots, \ell_i\}$. The set $proc(\lambda_{i,k})$ contains all the distinct cores associated to a path $\lambda_{i,k}$, whereas $v_{i,a}^p$ and $v_{i,z}^p$ represent the first and the last subtasks in $\lambda_{i,k}$ assigned to core p, respectively. We further define the length of a path $len(\lambda_{i,k})$ as the sum of the WCET of all its subtasks. Formally, $len(\lambda_{i,k}) = \sum_{\forall v_{i,\ell} \in \lambda_{i,k}} C_{i,\ell}$.

Definition 2 (Length). *The length $len(\tau_i)$ of a DAG task τ_i is the maximum length among all the λ_i paths, i.e. $len(\tau_i) = \max_{k=1}^{\ell_i}(len(\lambda_{i,k}))$.*

Note that, under partitioned scheduling, $len(\tau_i)$ may not represent the WCET of τ_i (also its WCRT in isolation) when the number of cores available is infinite. Indeed, the degree of parallelism is constrained by the subtask-to-core mapping and thus two independent subtasks assigned to the same core are forced to execute sequentially. Nevertheless, a typical necessary condition for the feasibility of τ_i is $len(\tau_i) \leq D_i$. Partitioned scheduling allows us to define further feasibility conditions. Hence, we introduce the notion of *p-workload*.

Definition 3 (p-Workload). *The worst-case workload of a DAG task τ_i on a core p, denoted by W_i^p, is the maximum processing time that any instance of τ_i requires from p, i.e. $W_i^p = \sum_{l=1}^{n_i}\{C_{i,\ell} \mid P_{i,\ell} = p\}$.*

The following additional relations must then hold for the feasibility of a DAG task system: $W_i^p \leq D_i$ and $\sum_{i=1}^{n} \frac{W_i^p}{T_i} \leq 1, \forall p \in [1, m]$.

Definition 4 (Workload). *The worst-case workload (or simply "workload") W_i of a DAG task τ_i is the sum of the WCET of*

[1]A transitive predecessor of $v_{i,b}$ is a subtask that must complete execution before a direct predecessor of $v_{i,b}$ can begin execution.

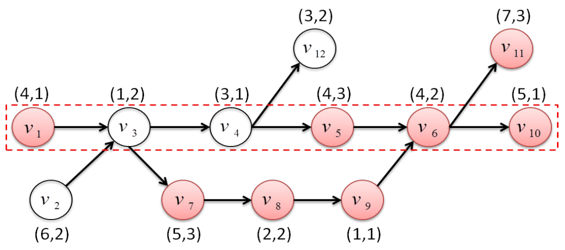


Fig. 1: An example DAG task with twelve subtasks. Each label indicates the WCET and the core affinity of the corresponding node, respectively.

all its subtasks irrespective of their mapping to the cores, i.e. $W_i = \sum_{l=1}^{n_i} C_{i,\ell}$.

Example. Fig. 1 illustrates our model for a DAG task τ_i comprised of twelve subtasks ($n_i = 12$) $V_i = \{v_1, \dots, v_{12}\}$ and twelve precedence constraints. For simplicity, hereinafter we omit the subscript i on the subtasks of τ_i. The label next to each node represents the 2-tuple $v_\ell = (C_\ell, P_\ell)$. For instance, subtask v_6 has a WCET of $C_6 = 4$ and is assigned to core $P_6 = 2$. There is a total of ten paths ($\ell_i = 10$) in the DAG. The critical path length of τ_i equals to $len(\tau_i) = 26$ and is found on the path $\lambda_{i,k} = (v_2, v_3, v_7, v_8, v_9, v_6, v_{11})$. Its workload is $W_i = 45$, whereas the maximum p-workload resides on core $p = 3$ with the value $W_i^3 = 16$, which results from the subset of subtasks $\{v_5, v_7, v_{11}\}$. Note that this subset imposes no sequencing of subtasks and thus is not necessarily a path on the DAG. To clarify, both v_5 and v_7 are transitive predecessors of v_{11}, but v_5 and v_7 are independent of each other.

IV. WCRT analysis of partitioned DAG tasks

In this section, we present a schedulability analysis for sporadic DAG tasks with constrained deadlines scheduled in a partitioned fashion with any fixed-priority scheduling algorithm. The schedulability analysis is based on the notion of per-path interference. Unlike its sequential counterpart, partitioned DAG tasks may allocate subtasks to different cores, potentially creating cross-core dependencies. Unless each path is entirely allocated to a single core, the traditional uniprocessor analysis for fixed-priority sequential tasks that relies on the concept of critical instant [20] cannot be applied on a per-path basis. Therefore, we introduce a novel WCRT analysis to cope with the notion of partitioned DAG tasks.

Let τ_i be the DAG task under analysis. For simplicity and clarity of presentation, we assume that all the higher priority tasks are sequential. That is, $\tau_j \overset{\text{def}}{=} (C_j, P_j, D_j, T_j, J_j)$, $\forall j < i$; meaning τ_j has a release jitter J_j, and each of its jobs can execute only on core P_j and for at most C_j time units. In Section VI, we generalize the analysis for the case where every task in the system is a DAG task.

We seek to derive an upper-bound on the WCRT of each path $\lambda_{i,k}$ of τ_i. To do so, we first introduce some definitions to characterize the different worst-case interference contributions.

Definition 5 (Inter-Task Interference)**.** *The inter-task interference $I_j(\lambda_{i,k})$ imposed by a higher priority task τ_j on the k^{th}path of task τ_i is the maximum cumulative time during which any subtask $v_\ell \in \lambda_{i,k}$ is ready but cannot execute because τ_j is executing on the same core.*

Definition 6 (Self-Interference)**.** *The self-interference $I_i(\lambda_{i,k})$ imposed by task τ_i on its own k^{th}path is the maximum cumulative time during which any subtask $v_\ell \in \lambda_{i,k}$ is ready to execute but cannot because access to the same core has been granted to subtasks of τ_i that do not belong to $\lambda_{i,k}$.*

Definitions 5 and 6 provide a characterization of the maximal overall interference exerted on a path $\lambda_{i,k}$ of task τ_i during the execution of any of its jobs. A response time equation for $\lambda_{i,k}$ can then be expressed as follows.

Theorem 1. *The worst-case response time of a path $\lambda_{i,k}$ of a partitioned DAG task τ_i is given by*

$$R(\lambda_{i,k}) = len(\lambda_{i,k}) + I_i(\lambda_{i,k}) + \sum_{\forall j<i} I_j(\lambda_{i,k}) \qquad (1)$$

Proof. Let r_a be the release time of the first subtask in the path $\lambda_{i,k}$. In the scheduling $[r_a, r_a + R(\lambda_{i,k})]$, the entire path requires at most $len(\lambda_{i,k})$ time units of execution. The total interference caused by self-interfering subtasks on $\lambda_{i,k}$ is $I_i(\lambda_{i,k})$ according to Definition 6. By Definition 5, the maximum interference exerted on $\lambda_{i,k}$ by each higher priority task (i.e., tasks τ_j such that $j < i$) is $I_j(\lambda_{i,k})$. The theorem follows by noting that the set of lower priority tasks cannot influence the schedule of τ_i in fixed-priority preemptive scheduling. □

To obtain the WCRT of τ_i we apply Equation 1 to each of its paths. The next corollary directly follows from Theorem 1.

Corollary 1. *The worst-case response time of a DAG task τ_i is given by*

$$R(\tau_i) = \max_{k=1}^{\ell_i} R(\lambda_{i,k}). \qquad (2)$$

As a direct consequence of Corollary 1, DAG task τ_i is deemed schedulable if $R(\tau_i) \le D_i$.

A. On the self-interference

Conceptually, the self-interference corresponds to the delay on the completion time of task τ_i caused by the concurrency among its own subtasks. Since the computing resources are typically scarce, subtasks of τ_i that could execute in parallel (i.e., they share no direct or transitive precedence constraint) will eventually contend for core-access, thus increasing the response time of τ_i itself. Under partitioned scheduling this phenomenon is easier to observe as the subtask-to-core assignment dictates which independent subtasks may interfere with each other.

On a particular path $\lambda_{i,k}$, $I_i(\lambda_{i,k})$ accounts for all the time intervals where any subtask $v_{\ell'} \in V_i \setminus \lambda_{i,k}$ is executing, while there is a ready subtask $v_\ell \in \lambda_{i,k}$ with pending work on the same core P_ℓ. Thus, if $P_{\ell'} \notin proc(\lambda_{i,k})$, then $v_{\ell'}$ can never interfere with $\lambda_{i,k}$. Contrary to the inter-task interference, only one instance of such self-interfering subtasks have to be accounted in $I_i(\lambda_{i,k})$ because they belong to the same job of τ_i[2]. The absence of individual priorities assigned to the

[2]In a hard real-time constrained-deadline system, there is no interference between different jobs of the same task if no deadlines are missed.

subtasks, together with variability in their execution times, makes self-interference a problem mutual to all the paths of τ_i. That is, if a path A interferes with a path B, then it is also possible to construct a scenario where path B interferes with path A. Furthermore, we observe that when a subtask $v_{\ell'} \in V_i \setminus \lambda_{i,k}$ shares no independent constraints with any subtask $v_\ell \in \lambda_{i,k}$, where $P_\ell = P_{\ell'}$, $v_{\ell'}$ cannot interfere with $\lambda_{i,k}$. This allows us to derive a less conservative upper-bound on $I_i(\lambda_{i,k})$ comparatively to the global approaches.

Lemma 1. *For any partitioned constrained-deadline DAG task τ_i partitioned on m cores, an upper-bound on the interfering workload imposed by τ_i on its path $\lambda_{i,k}$ is given by*

$$I_i(\lambda_{i,k}) \leq \sum_{\forall p \,\in\, proc(\lambda_{i,k})} W_i^p - len(\lambda_{i,k}) - \sum_{\forall v_\ell \,\in\, \Theta_{i,k}} C_\ell, \quad (3)$$

where $\Theta_{i,k}$ is the set of subtasks of τ_i that cannot interfere with $\lambda_{i,k}$ and is defined as

$$\Theta_{i,k} \overset{\text{def}}{=} \bigcup_{\forall p \,\in\, proc(\lambda_{i,k})} pred(v_a^p, p) \cup succ(v_z^p, p). \quad (4)$$

Proof. By definition of a constrained-deadline task, two instances of τ_i cannot interfere with each other when D_i is met. Therefore, the maximum interference that τ_i can impose on its path $\lambda_{i,k}$ is upper-bounded by the sum of the WCET of all subtasks assigned to a core $p \in proc(\lambda_{i,k})$ excluding the subtasks in $\lambda_{i,k}$, i.e. $I_i(\lambda_{i,k}) \leq \sum_{\forall p \,\in\, proc(\lambda_{i,k})} W_i^p - len(\lambda_{i,k})$. Any subtask v_ℓ that is a successor, either directly or transitively, of v_z^p (i.e., the last subtask of $\lambda_{i,k}$ assigned to core p), where $p = P_\ell$, cannot interfere with $\lambda_{i,k}$ because v_ℓ becomes ready only after all subtasks in $\lambda_{i,k}$ assigned to P_ℓ complete. Similarly, any subtask v_ℓ that is a predecessor, either directly or transitively, of v_a^p (i.e., the first subtask of $\lambda_{i,k}$ assigned to core p), where $p = P_\ell$, can be safely discarded because any delay caused by v_ℓ on the start of v_a^p will be accounted in any path $\lambda_{i,k'}$, where $k' \neq k$ and $\{v_\ell, v_a^p\} \subseteq \lambda_{i,k'}$ (from Equation 2). As given by Equation 4, the set $\Theta_{i,k}$ contains all those non-interfering subtasks. Hence, $I_i(\lambda_{i,k}) \leq \sum_{\forall p \,\in\, proc(\lambda_{i,k})} W_i^p - len(\lambda_{i,k}) - \sum_{\forall v_\ell \,\in\, \Theta_{i,k}} C_\ell$, which concludes the proof. □

Henceforth, let $self(\lambda_{i,k})$ denote the set of self-interfering subtasks with $\lambda_{i,k}$.

B. On the inter-task interference

The inter-task interference $I_j(\lambda_{i,k})$ accounts for all the time intervals during which a subtask $v_\ell \in \lambda_{i,k}$ is ready but it cannot execute since higher priority task τ_j is holding the processor. Hence, τ_j may interfere with multiple subtasks in $\lambda_{i,k}$. Despite τ_j being any task with higher priority than τ_i, if $P_j \notin proc(\lambda_{i,k})$, then τ_j cannot interfere with $\lambda_{i,k}$. We denote by $hp(\lambda_{i,k})$ the set of higher priority tasks that can effectively interfere with $\lambda_{i,k}$.

Although in global multiprocessor scheduling one must consider carry-in jobs as part of the individual interference contributions [18], it is not the case for the fixed-priority scheduling analysis of a partitioned DAG task when the higher priority tasks are sequential and also partitioned as the problem boils down to single core scheduling. In this context, $I_j(\lambda_{i,k})$ is a function of the maximum number of interfering jobs released by τ_j in a scheduling window $[r_a^p, f_z^p)$, where r_a^p is the release time of subtask $v_a^p \in \lambda_{i,k}$ and f_z^p is the completion time of subtask $v_z^p \in \lambda_{i,k}$, as τ_j cannot interfere with subtasks on a core different than P_j. No job released by τ_j at any instant prior to r_a^p interferes with $\lambda_{i,k}$ because otherwise there would exist a valid scheduling window $[r_a^p - t, f_z^p)$ that would increase the response time.

A simple solution to upper-bound $I_j(\lambda_{i,k})$ is to assume that τ_j is allowed to release jobs synchronously with every subtask $v_\ell \in \lambda_{i,k}$ such that $P_j = P_\ell$. Subsequent job releases are then separated by T_j time units. A similar technique to this independent worst-case scenario for each subtask in $\lambda_{i,k}$ was adopted in [6]. Clearly, it is not always possible for any τ_j to interfere with all the workload of $\lambda_{i,k}$ assigned to P_j, thus this technique is often overly pessimistic.

Unfortunately, finding tight upper-bounds on the interfering workload of the higher priority tasks is difficult. The challenge comes from the fact that the workload of $\lambda_{i,k}$ on a core p may not be continuous, as some of the intermediate subtasks are assigned to different cores. As a result, cross-core dependencies on the execution flow of the path exist. The length of these discontinuous intervals is not fixed since the release of a transitive successor on p depends on the response time of the intermediate subtasks mapped to other cores. Therefore, $I_j(\lambda_{i,k})$ is not only a function of the response time of the subtasks of path $\lambda_{i,k}$ on core $P_j = p$, but it also has to account for the gaps within the different parts of the workload, and for the relation between the duration of the gaps and the response time of subtasks on any core $p \neq P_j$.

To overcome this problem, we propose in the next section an alternative technique to Equation 1, for computing an upper-bound on the response time of any path $\lambda_{i,k}$, which is based on the self-suspending tasks theory.

V. Response time of a path $\lambda_{i,k}$

In this section, we explain how to compute an upper-bound on the worst-case response time of each path $\lambda_{i,k}$ of task τ_i. To capture the worst-case interference suffered by a path on the different cores to which it is mapped, we model a path as a set of self-suspending tasks. More precisely, one self-suspending task for each core. For simplicity, hereinafter, we assume that any two consecutive subtasks assigned to the same core are merged into a single subtask with a WCET equal to the sum of their individual WCETs. Because this section refers to only one specific path $\lambda_{i,k}$, for the sake of notation conciseness, we let λ denote that path $\lambda_{i,k}$.

A. Intuition

In the literature, a self-suspending task is often described as an interleaved sequence of execution and suspension regions, where an execution region is a portion of a sequential task that needs to be processed, and a suspension region corresponds to a period of time during which the task voluntarily yields the processor to perform a remote operation. The suspension regions are assumed to have given bounded durations.

Regarding partitioned DAG tasks, we observe that the existence of precedence constraints between subtasks assigned

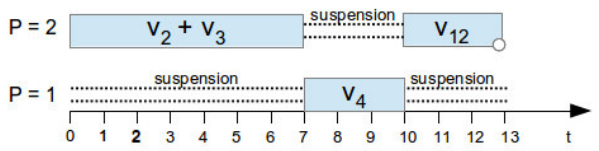


Fig. 2: An example of partitioned schedule for the path of Fig. 1 formed by the light nodes.

to different cores leads to a similar behavior. This is easier to see on a path, since there is a direct precedence constraint between every two consecutive subtasks. As depicted in Fig. 2, consider an example of partitioned schedule for the path highlighted in the DAG of Fig. 1. Although only subtask v_4 is assigned to core $p = 1$, it cannot start executing before the joint subtask $v_1 + v_3$ completes execution on core $p = 2$. In turn, the execution of v_4 delays the release of the last portion of workload assigned to core $p = 2$ (i.e. subtask v_7), creating an intermediate interval of 3 time units where core $p = 2$ is free to execute lower priority tasks. Such time intervals can be seen as suspensions within the path on a certain core p.

In this sense, a path λ can be modeled as a set of sporadic self-suspending tasks, one for each core reached by the path, i.e. $\forall p \in proc(\lambda)$. The trick is to treat the subtasks of λ assigned to the current core under analysis as execution regions and the response time of all the remaining subtasks as suspension regions. The problem of computing the response time of λ on a multicore platform becomes then equivalent to the analysis of $|proc(\lambda)|$ self-suspending tasks in a uniprocessor system. However, unlike previous works, the duration of the suspension regions are not known beforehand as they are in fact computations to be executed on different cores.

Subtasks assigned to a core $p' \neq p$ and released before the first execution region or after the last execution region of a self-suspending task formed on core p must not be consider as part of its suspension regions, since they do not contribute to an increase on the interfering workload[3]. For instance, the suspensions on core $p = 1$ in Fig. 2 do not influence the response time of v_4. Every other subtask assigned to a core $p' \neq p$ is henceforth called "remote subtask".

B. The self-suspending task model

For ease of understanding, we start with a generic model that closely relates to the ones considered in the literature. Let τ^p denote the self-suspending task formed by a path λ on core p. Each self-suspending task τ^p, $\forall p \in proc(\lambda)$, consists of $q \geq 1$ *execution regions* and $q - 1$ *suspension regions* such that any two consecutive execution regions are separated by a suspension region. Formally, $\tau^p \overset{\text{def}}{=} \{(C_1^p, S_1^p, C_2^p, \ldots, S_{q-1}^p, C_q^p), S^{p,ub}\}$, where C_h^p is the WCET of the h^{th} execution region of τ^p, while an upper-bound on the duration of the h^{th} suspension region of τ^p is given by S_h^p. The parameter $S^{p,ub}$ denotes an upper-bound on the overall suspension time.

[3]Such suspensions are relevant only when the self-suspending task is part of the interfering workload, which is not the case here.

Note that $S^{p,ub}$ is not necessarily equal to the maximum cumulative suspension time. That is, $S^{p,ub} \leq \sum_{h=1}^{q-1} S_h^p$. While it is easy to see that each C_h^p corresponds to the WCET of the h^{th} subtask of λ assigned to core p, the values of each S_h^p and $S^{p,ub}$ cannot be directly derived from λ and must therefore be computed. We show next how these values can be expressed as functions of the WCRT of the remote subtasks of τ^p.

Additionally, we represent by E_h^p the h^{th} execution region of τ^p, $\forall h \in [1, q]$. A sequential task is a self-suspending task with no suspension regions. In this particular case, τ^p is represented as: $\tau^p \overset{\text{def}}{=} \{(C_1^p), 0\}$.

C. Methodology

The purpose of the above model is to not tie the analysis of a partitioned DAG task to a particular work on self-suspending tasks. Thus, any existing uniprocessor timing analysis for fixed-priority sporadic self-suspending tasks can be used to derive the WCRT of a self-suspending task τ^p. As the WCRT of each τ^p encompasses the WCRT of all the subtasks assigned to core p, the WCRT of a path λ can be expressed by its self-suspending tasks. Consequently, Eq. 1 is rewritten as follows.

Theorem 2. *The WCRT of any path λ is upper-bounded by*

$$R(\lambda) \leq \sum_{\forall p \in proc(\lambda)} R(\tau^p) - S^{p,ub} \tag{5}$$

Proof. Each self-suspending task τ^p models the worst-case request of the path λ on core p. By definition, $R(\tau^p)$ upper-bounds the cumulative response time of its execution and suspension regions. Thus $R(\tau^p)$ also upper-bounds the sum of the response time of the subtasks of λ assigned to core p. By summing the WCRT of each τ^p, $\forall p \in proc(\lambda)$, we therefore upper-bound the sum of the response time of all the subtasks in λ. Furthermore, because by definition of τ^p the suspension time of a task τ^p corresponds to the processing time of λ on other cores than p, the suspension time $S^{p,ub}$ should not be considered as part of $R(\tau^p)$ as it is already accounted by the other self-suspending tasks $\tau^{p'}$, where $p' \neq p$. $\square$

An important aspect to consider is that the self-suspending tasks depend on each other through the suspension regions. Bounds on the suspension regions are necessary to analyze the WCRT of the execution regions, but the suspension regions are indeed execution regions on other cores. In the following we explain how to break this circular dependency by capturing the worst-case behavior of the remote subtasks of τ^p.

A well-known result from the sporadic self-suspending tasks theory is that larger suspensions cannot lead to a decrease in the interference suffered by the task under analysis. Therefore, we are interested in finding the WCRT of all the remote subtasks of τ^p as a way to characterize the worst-case request of τ^p. We first present how $S^{p,ub}$ can be computed.

Whenever multiple remote subtasks of τ^p are assigned to the same core p', assuming an independent WCRT for each one of them is overly pessimistic. In fact, they form an inner self-suspending task within τ^p. Let τ_{ss} denote that inner self-suspending task. Then, the WCRT of all such remote subtasks on p' together is given by $R(\tau_{ss}) - S_{ss}^{ub}$.

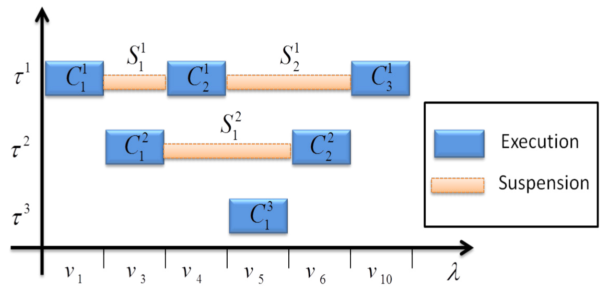


Fig. 3: Overview of the path λ highlighted in Fig. 1 as a set of self-suspending tasks.

A special case happens when there is only one remote subtask of τ^p allocated to core $P_l \neq p$. Let v_ℓ be that subtask. If no other remote subtask of τ^p is assigned to P_l, then the worst-case duration of the suspension region generated by v_ℓ is given by the worst-case response time $R(v_\ell)$ of v_ℓ. Subtask v_ℓ can then be considered an independent sequential task for which the critical instant in uniprocessor holds.

Let $R(\tau_{ss}^o)$ denote the upper-bound on the WCRT of the remote subtasks of τ^p assigned to a core o. Then, the maximum overall suspension time experienced by τ^p is

$$S^{p,ub} = \sum_{\forall o \in proc(\lambda), o \neq p} R(\tau_{ss}^o) - S_{ss}^{o,ub}. \tag{6}$$

Although parameter $S^{p,ub}$ provides no information about the relation between the different suspension regions, it can seamlessly serve as input to any self-suspending analysis that disregard the placement and duration of each suspension region specifically.

We now present how to compute an upper-bound on S_h^p, $\forall h \in [1, q-1]$. A simple solution is to compute an independent WCRT for each of the remote subtasks of τ^p and sum the resulting values of those that belong to the same suspension region. Thus, if the remote subtasks v_ℓ and $v_{\ell+1}$ separate the execution regions E_1^p and E_2^p, then $S_1^p = R(v_\ell) + R(v_{\ell+1})$ (similarly to [6]). The pessimism can be reduced by considering the fact that if a inner self-suspending task τ_{ss} resides inside a single suspension region of τ^p, the cumulative WCRT of the remote subtasks of τ^p on p' that correspond to execution regions in τ_{ss} can be replaced by $R(\tau_{ss}) - S_{ss}^{ub}$. Assume that only remote subtask v_ℓ separates such remote subtasks and $p \neq P_\ell \neq p'$. Accordingly, $S_h^p = R(\tau_{ss}) - S_{ss}^{ub} + R(v_\ell)$.

Example. Fig. 3 depicts how each core perceives the path λ of task τ_i highlighted in Fig. 1. We describe through this example how to characterize each self-suspending task τ^p for that particular path. On core $p = 3$, all the remote subtasks appear before and after the execution region E_1^3 constituted of v_5, so τ^3 is a sequential task with $C_1^3 = C_5$. On core $p = 2$, τ^2 has $q = 2$ execution regions with $C_1^2 = C_3$ and $C_2^2 = C_6$ and a single suspension region comprised of two remote subtasks. The maximum duration of the suspension region is given by the independent upper-bounds on v_4 and v_5, i.e. $S_1^2 = R(v_4) + R(v_5)$. Since there is only one suspension region, $S^{2,ub} = S_1^2$. Finally, τ^1 has $q = 3$ execution regions with $C_1^1 = C_1$, $C_2^1 = C_4$ and $C_3^1 = C_{10}$, and two suspension regions for which $S_1^1 = R(v_3)$ and $S_2^1 = R(v_5) + R(v_6)$. Note, however, that suspending subtasks v_3 and v_6 form in fact τ^2. Thus, $S^{1,ub} = R(\tau^2) - S^{2,ub} + R(v_5)$.

Given the above relations, it becomes clear that to capture the worst-case request of a self-suspending task τ^p many inner self-suspending tasks must be characterized and analyzed. This implies that an effective order of computations in necessary to drive the whole algorithm.

D. Unfolding the path

To overcome the dependency problem, we propose an algorithm that recursively divides a path into smaller ones, creating a tree of subpaths which represent self-suspending tasks. The tree reflects the hierarchy of dependencies. When a leaf is reached, the corresponding task has no suspension regions (i.e., it is a sequential task), thus its WCRT does not depend on anything else other than the interfering workload on that core and can be computed immediately. The computed values are then back propagated to the self-suspending tasks on the upper levels, so that their suspension time is no longer unknown. After setting the appropriate bounds on the suspension regions, the WCRT of the execution regions of the next self-suspending task is derived. The process continues level-by-level until the root is revisited. At this point, the WCRT of the original path can be computed thanks to Theorem 2.

Algorithm 1 shows the pseudo-code of the recursive algorithm for unfolding any path λ and computing the required WCRTs. It takes as input a path λ_{ss}, and both the set of higher priority tasks $hp(\lambda)$ and the set of self-interfering subtasks $self(\lambda)$ regarding the path λ. Initially, $\lambda_{ss} = \lambda$. The algorithm starts by finding the number of subtasks in the path λ_{ss} (line 2). If the path spawns over more than one core (lines 10-30), there are self-suspending tasks with suspension regions to be identified. Here, we consider two cases.

First (lines 11-16), if both the first and the last subtask (v^{1st} and v^{last} respectively) of λ_{ss} reside on the same core (let p be that core), then λ_{ss} constitutes one of the self-suspending tasks[4] on core p. However, its response time cannot be computed straight away because the bounds on its suspension regions are not known yet. Thus, we exclude v^{1st} and v^{last} from λ_{ss} and invoke the algorithm for the resulting subpath. The second case deals with paths that start and end on different cores (lines 17-29). In this situation, we just split the path λ_{ss} in two subpaths to be analyzed: i) from the first subtask of λ_{ss} on the same core as v^{last} to v^{last} (lines 18-26), and ii) λ_{ss} except v^{last} (lines 27-28).

The recursion on a path stops when there is only one subtask in λ_{ss}. The WCRT of a single subtask can then be computed by adding the self-interfering workload to the traditional equation for fixed-priority sequential tasks in uniprocessors. That is, the WCRT $R(\lambda_{ss})$ when λ_{ss} contains only one subtask is given by Eq. 7 and reflected in line 8 of Algorithm 1.

$$R(\lambda_{ss}) = C^{1st} + \sum_{\forall \tau_j \in hp(\lambda_{ss})} \left\lceil \frac{R(\lambda_{ss}) + J_j}{T_j} \right\rceil \times C_j + \sum_{\forall v_\ell \in self(\lambda_{ss})} C_\ell \tag{7}$$

[4]Note that a self-suspending task τ^p may be decomposed into multiple inner self-suspending tasks, some of which constituting remote subtasks for a self-suspending task $\tau^{p'}$ where $p \neq p'$.

As soon as all the subpaths of a self-suspending task τ_{ss} on p have been analyzed, the bound on the total duration of its suspensions S_{ss}^{ub} is given by the sum of the WCRT of its inner self-suspending tasks that are not on p (as discussed in Section V-C). All of these values have already been computed and available in the matrix RTs returned by Algorithm 1. Every $C_{ss,h}$ is assigned with the WCET of the h^{th} subtask in λ_{ss} on p. Apart from the overall bound on the suspension time, an individual upper-bound $S_{ss,h}$ on each suspension region is also required. While parsing the path λ_{ss}, if the WCRT of a remote subtask is missing in RTs, we apply Eq. 7 to that particular subtask. This allows us to compose the values $S_{ss,h}$ according to the relations of the remote subtasks with each suspension region. All such operations are performed by the function $setSuspendingTask$ in line 14. With all the parameters in τ_{ss} defined, the WCRT of the self-suspending task is compute at line 15. Details about the timing analysis are provided in the subsection below. Finally, the procedure is repeated until all the self-suspending tasks are analyzed.

Applying Algorithm 1 to a path λ guarantees that the WCRT of every self-suspending task τ^p defined from λ has been correctly computed.

Algorithm 1: Computation of the WCRT of each self-suspending task in the path λ

```
1  Function PathAnalysis ($\lambda_{ss}$, $hp(\lambda)$, $self(\lambda)$) is
       Inputs : $\lambda_{ss}$ - (sub) path under analysis
                $hp(\lambda)$ - set of higher priority tasks w.r.t. path $\lambda$
                $self(\lambda)$ - set of self-interfering nodes w.r.t path $\lambda$
       Output: $RTs$ - matrix $n_i \times n_i$ that stores the computed response
                time from a subtask $v_a$ to another subtask $v_b$
2      $size \leftarrow |\lambda_{ss}|$;
3      $v^{1st} \leftarrow$ first subtask in $\lambda_{ss}$;
4      $v^{last} \leftarrow$ last subtask in $\lambda_{ss}$;
5      $hp(\lambda_{ss}) \leftarrow \bigcup_{\forall \tau_j \in hp(\lambda)} \{\tau_j \mid P_j = P^{1st}\}$;
6      $self(\lambda_{ss}) \leftarrow \bigcup_{\forall v_\ell \in self(\lambda)} \{v_\ell \mid : P_\ell = P^{1st}\}$;
7      if $size = 1$ then
8          $R(\lambda_{ss}) = C^{1st} + \sum_{\forall \tau_j \in hp(\lambda_{ss})} \lceil \frac{R(\lambda_{ss})+J_j}{T_j} \rceil \times C_j + \sum_{\forall v_\ell \in self(\lambda_{ss})} C_\ell$;
9          $RTs(v^{1st}, v^{last}) \leftarrow R(\lambda_{ss})$;
10     else
11         if $P^{1st} = P^{last}$ then
12             $\lambda_{ss}^{sub} \leftarrow \lambda_{ss} \setminus \{v^{1st}, v^{last}\}$;
13             $PathAnalysis(\lambda_{ss}^{sub},\ hp(\lambda),\ self(\lambda))$;
14             $\tau_{ss} \leftarrow setSuspendingTask()$;
15             $R(\tau_{ss}) \leftarrow ssRT(\tau_{ss}, hp(\lambda_{ss}), self(\lambda_{ss}))$;
16             $RTs(v^{first}, v^{last}) \leftarrow R(\tau_{ss})$;
17         else
18             $\lambda_{ss}^{sub} \leftarrow \lambda_{ss}$;
19             foreach $v_\ell \in \lambda_{ss}$ do
20                 if $P_\ell \neq P^{last}$ then
21                     $\lambda_{ss}^{sub} \leftarrow \lambda_{ss}^{sub} \setminus \{v_\ell\}$;
22                 else
23                     break;
24                 end
25             end
26             $PathAnalysis(\lambda_{ss}^{sub},\ hp(\lambda),\ self(\lambda))$;
27             $\lambda_{ss}^{sub} \leftarrow \lambda_{ss} \setminus \{v^{last}\}$;
28             $PathAnalysis(\lambda_{ss}^{sub},\ hp(\lambda),\ self(\lambda))$;
29         end
30     end
31     return $RTs$;
32 end
```

E. WCRT of a self-suspending task

We now show how three different state-of-the-art response time analyses [21], [22] for sporadic self-suspending tasks can be extended to cope with both the dependent suspension regions and the self-interfering workload discussed in the previous sections. The worst-case release pattern for the higher priority tasks follows directly from the results of these analyses. Without loss of generality, we consider the WCRT of a self-suspending task τ^p. We start by proving the worst-case release pattern for the self-interfering subtasks in $self(\tau^p)$.

Lemma 2. *The contribution of $self(\tau^p)$ to the WCRT of a self-suspending task τ^p is upper-bounded by releasing a single instance of each self-interfering subtask in $self(\tau^p)$ synchronously with any one execution region E_h^p of τ^p.*

Proof. Following Lemma 1, a self-interfering subtask $v_\ell \in self(\tau^p)$ can delay the execution of a self-suspending task τ^p at most once for C_ℓ time units. Since all subtasks in τ_i have the same priority, they cannot preempt each other. Thus, v_ℓ cannot execute when an execution region of τ^p is active. The maximum interference imposed by $self(\tau^p)$ on τ^p happens then when each $v_\ell \in self(\tau^p)$ is released synchronously with an execution region E_h^p. Although such self-interference is upper-bounded by $\sum_{v_\ell \in self(\tau^p)} C_\ell$ as a whole, the number of higher priority interfering jobs is influenced by the exact placement of each self-interfering subtask, as assuming that v_ℓ interferes with the execution region E_h^p is equivalent to increase C_h^p by C_ℓ time units. In general, enlarging different execution regions leads to different response times. Hence, allowing each of these self-interfering subtasks to be released with any one execution region of τ^p captures the worst-case contribution of $self(\tau^p)$ to $R(\tau^p)$. □

Suspension-oblivious analyses [21] treat suspension regions as part of the computations to be processed, making suspensions subject to the same sources of interference than the execution regions. In this sense, the entire self-suspending task is model as a single sequential task. Consequently, an upper-bound on the WCRT of τ^p is found when the overall suspension time is the largest (i.e. $S^{p,ub}$) and the self-interfering subtasks are released synchronously with E_1^p. That is,

$$R(\tau^p) = \sum_{h=1}^{q} C_h^p + S^{p,ub} + \sum_{\forall \tau_j \in hp(\tau^p)} \lceil \frac{R(\tau^p) + J_j}{T_j} \rceil \times C_j + \sum_{\forall v_\ell \in self(\tau^p)} C_\ell \quad (8)$$

The simplest suspension-aware analysis [21] focus on upper-bounding the WCRT of each execution region independently. That is, the problem is reduced to a set of smaller sequential tasks by assuming that all the interfering workload releases jobs synchronously with each and every execution region. In this case, the self-interfering subtasks must be accounted once in each execution region E_h^p. Moreover, the suspension time has no influence on the interference generated. By using Eq. 7 to compute each $R(E_h^p)$, the WCRT of τ^p is given by

$$R(\tau^p) = S^{p,ub} + \sum_{h=1}^{q} R(E_h^p) \quad (9)$$

Both of the aforementioned tests run in pseudo-polynomial time but are substantially pessimistic. The pessimism is further aggravated because multiple self-suspending tasks need to be analyzed to bound the suspensions regions of τ^p. Nelissen et al. [22] proposed a MILP formulation that finds tight upper-bounds (the best known) on the WCRT of a sporadic self-suspending task with multiple suspension regions. The formulation exploits the duration of the suspension regions to accurately upper-bound the number of jobs released by each higher priority task in each execution region. Therefore, a weak characterization of the relation between the different suspension regions, and also their own bounds, compromises the quality of the solution. As it is implicit in the relation $S^{p,ub} \leq \sum_{h=1}^{q-1} S_h^p$, some upper-bounds on the duration of the suspension regions of τ^p are over-estimated. Hence we discuss the limitations of our generic model and propose a more robust one that adheres to such formulation.

Consider the following example: τ^p has two suspension regions, each one of them comprised of a single remote subtask assigned to the same core; let v_ℓ and $v_{\ell+1}$ represent those remote subtasks, while τ_{ss} denotes the inner self-suspending task formed by them. If $R(\tau_{ss}) - S_{ss}^{ub} < R(v_\ell) + R(v_{\ell+1})$, then the first suspension region plus the second suspension region cannot exceed $R(\tau_{ss}) - S_{ss}^{ub}$. Consequently, one must find the trade off that satisfies $R(\tau_{ss}) - S_{ss}^{ub}$, while still representing a worst-case suspension pattern for τ^p.

To address this issue, let S_h^p denote instead the h^{th} suspension region of τ^p and be characterized by the 2-tuple $(S_h^{p,lb}, S_h^{p,ub})$, $\forall h \in [1, q-1]$. The parameter $S_h^{p,lb}$ is a lower-bound on the duration of the suspension region S_h^p, whereas $S_h^{p,ub}$ is an upper-bound. We now prove the values for the individual lower-bounds.

Lemma 3. *Let V_h^p denote the set of remote subtasks within E_h^p and E_{h+1}^p of τ^p. A lower-bound on the suspension region S_h^p is given by $S_h^{p,lb} = \sum_{v_\ell \in V_h^p} C_\ell$.*

Proof. The WCRT of τ^p is found when the response time of its remote subtasks is maximized. We must therefore prove that the WCRT of each inner self-suspending tasks τ_{ss}^o ($\forall o \in proc(\lambda)$, $o \neq p$) within τ_p cannot be met if a remote subtask $v_\ell \in V_h^p$ executes for less than C_ℓ time units. The proof is by contradiction. Consider a release pattern σ for any inner self-suspending task τ_{ss}^o that maximizes $R(\tau_{ss}^o)$ but where $R(E_{ss,h}^o) < C_{ss,h}^o$. The execution region $E_{ss_h}^o$ of τ_{ss}^o corresponds to a remote subtask $v_\ell \in V_h^p$. If $E_{ss,h}^o$ executes for its WCET $C_{ss,h}^o = C_\ell$, an equivalent release pattern σ' can be obtained by delaying each subsequent interfering job release by $C_{ss,h}^o - R(E_{ss,h}^o)$ time units. This means that the entire window $[R(E_{ss,h}^o), R(\tau_{ss}^o)]$ in σ is repeated after the initial $C_{ss,h}^o$ time units in σ'. Clearly, $R(\tau_{ss}^o)$ is not the worst-case response time of τ_{ss}^o which invalidates the hypothesis. □

If an inner self-suspending task τ_{ss}^o is comprised of a single remote subtask $v_\ell \in V_h^p$ (i.e., τ_{ss}^o is sequential), than the value of $S_h^{p,lb}$ can be improved by replacing the contribution of that particular remote subtask with $R(v_\ell)$ instead of C_ℓ. Irrespectively of the type of the inner self-suspending tasks, the upper-bound $S_h^{p,ub}$ is computed as explained in Section V-C. Based on these individual bounds on the suspension regions, the following property holds.

Property 1. *Let $R(\tau_{ss,h}^o)$ denote the total WCRT of the remote subtasks within E_h^p and E_{h+2}^p of τ^p assigned to core o, $\forall o \in proc(\lambda)$, $o \neq p$. Admitting any solution, such that (1) $S_h^p + S_{h+1}^p \leq \sum_{\forall o \in proc(\lambda), o \neq p} R(\tau_{ss,h}^o)$, (2) $S_h^{p,lb} \leq S_h^p \leq S_h^{p,ub}$, and (3) $S_{h+1}^{p,lb} \leq S_{h+1} \leq S_{h+1}^{p,ub}$, upper-bounds the worst-case suspension behavior of the suspension regions S_h^p and S_{h+1}^p of τ^p.*

The reasoning behind Property 1 is that, in the general case, no technique exists yet to deem how the suspension time should be distributed between the suspension regions so that $R(\tau^p)$ is maximized. Hence, all the possible combinations should be considered. Property 1 must be applied to every sequence of suspensions regions in which an inner self-suspending task (not sequential) of τ^p can be identified. We denote by ψ^p the set of constraints that restrict the duration of multiple suspension regions of τ^p together (i.e., constraints as $S_h^p + S_{h+1}^p \leq \sum_{\forall o \in proc(\lambda), o \neq p} R(\tau_{ss,h}^o)$). Note that an optimization problem is clearly adequate to address the complexity exposed by Lemma 2 and Property 1. Therefore, we describe how to integrate them in the formulation of [22].

In the extended MILP formulation, the duration of each suspension region in τ^p is a real variable denoted S_h, while $Y_{\ell,h}$ is a binary variable that indicates whether ($Y_{\ell,h} = 1$) or not ($Y_{\ell,h} = 0$) a self-interfering subtask $v_\ell \in self(\tau^p)$ is released synchronously with the execution region E_h^p. Lemma 2 is formalized by Constraint 10

$$\forall v_\ell \in self(\tau^p) \ : \ \sum_{h=1}^{q} Y_{\ell,h} \leq 1, \tag{10}$$

which can then be integrated in the WCRT computation of each execution region as in Constraint 11, where $NI_{j,h}$ is the number of interfering jobs of τ_j in E_h^p.

$$\forall E_h^p \in \tau^p \ :$$

$$R(E_h^p) = C_h^p + \sum_{\tau_j \in hp(\tau^p)} NI_{j,h} \times C_j + \sum_{v_\ell \in self(\tau^p)} Y_{\ell,h} \times C_\ell. \tag{11}$$

That is, each self-interfering subtask interferes with exactly one execution region for its WCET. The individual bounds on the duration of each suspension region are expressed by Constraint 12.

$$\forall h \in [1, q-1] \ : \qquad S_h^{p,lb} \leq S_h \leq S_h^{p,ub} \tag{12}$$

Constraint 13 enforces that the sum of the suspension regions cannot exceed the upper-bound on the overall suspension time. For a more accurate analysis, Constraint 13 should be replaced with all the global constraints defined in ψ^p.

$$\sum_{h=1}^{q-1} S_h \leq S^{p,ub} \tag{13}$$

VI. Higher Priority DAG tasks

In this section, we extend our analysis to cope with multiple partitioned DAG tasks interfering with each other. As the WCRT of a DAG task τ_i is ultimately derived through a collection of self-suspending tasks τ_i^p, we restrict our attention to the worst-case interference impose by the higher priority tasks on any τ_i^p. Let V_j^p denote the set of subtasks of the higher priority DAG task τ_j assigned to core p. We prove below that each τ_j can safely be replaced in the response time analysis of τ_i^p by a set $V_j^{p'}$ of n_j^p sequential tasks, where $n_j^p = |V_j^p|$.

Let a sequential task $\tau_{j,o} \in V_j^{p'}$ correspond to the subtask $v_o \in V_j^p$. The task $\tau j, o$ upper-bounds the worst-case request of the subtask v_o, $\forall o \in [1, n_j^p]$, and is defined as $\tau_{j,o} \stackrel{\text{def}}{=} \langle C_{j,o}, D_{j,o}, T_{j,o}, J_{j,o} \rangle$. The worst-case execution time $C_{j,o}$ of $\tau_{j,o}$ is given by the WCET of $v_o \in V_j^p$, that is, $C_{j,o} \stackrel{\text{def}}{=} C_o$. Both the deadline $D_{j,o}$ and the period $T_{j,o}$ are inherited from τ_j. The parameter $J_{j,o}$ denotes the release jitter of $\tau_{j,o}$ and is defined as the difference between the WCRT of τ_j and the WCET of $v_o \in V_j^p$. Formally, $J_{j,o} \stackrel{\text{def}}{=} R(\tau_j) - C_o$. This transformation eliminates the dependencies regarding the interfering workload of τ_j towards τ_i^p by assuming that each subtask of τ_j assigned to core p is released independently. A similar method was already proposed in [6]. Note that the method in [6] could be used here too. The results would in fact be more precise, but at the cost of additional computation complexity.

Theorem 3. *The interference exerted by a DAG task $\tau_j \in hp(\tau_i)$ on a self-suspending task τ_i^p is upper-bounded by the sum of the interferences imposed on τ_i^p by each sequential task $\tau_{j,o} \in V_j^{p'}$, where $\tau_{j,o} \stackrel{\text{def}}{=} \langle C_{j,o}, D_{j,o}, T_{j,o}, J_{j,o} \rangle$ as defined above.*

Proof. The interference exerted by τ_j on τ_i^p is equal to the sum of the interference caused by each of the subtasks $v_o \in V_j^p$. We must therefore prove that the interference caused by each task $\tau_{j,o} \in V_j^{p'}$ upper-bounds the interference generated by each $v_o \in V_j^p$. The proof is by contradiction. Let us assume that v_o causes more interference than $\tau_{j,o}$. There might be only two reasons for this to be true: (i) a job released by v_o creates more interference than a job released by $\tau_{j,o}$, or (ii) v_o releases more jobs than $\tau_{j,o}$ in a given time window.

Since $\tau_{j,o}$ and v_o are both non-self-suspending and the WCET of $\tau_{j,o}$ is equal to the WCET of v_o, (i) cannot be true. As the minimum inter-arrival time of v_o is identical to that of its corresponding sequential task in $V_j^{p'}$, only their jitters may cause (ii) to be true. Now, let us compute the maximum jitter that can be experienced by the subtask v_o. Let a_j denote the arrival time of any job of τ_j. Since $R(\tau_j)$ assumes that each subtask of τ_j executes for its WCET, it means that a subtask v_o of τ_j cannot start executing later than $R(\tau_j) - C_o$ after a_j (otherwise it would complete later than $a_j + R(\tau_j)$ and $R(\tau_j)$ would not be the WCRT of τ_j). The release jitter of v_o is therefore upper-bounded by $J_o \stackrel{\text{def}}{=} R(\tau_j) - C_o$. This contradicts (ii) and hence proves the lemma. □

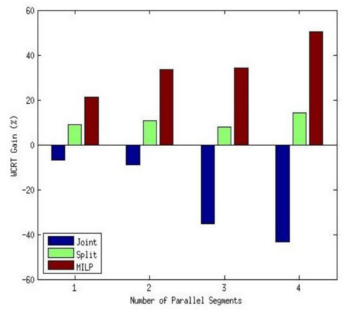
VII. Experiments

In this section, we describe experiments conducted through randomly generated task sets to evaluate (i) the performance of the different tests proposed in Section V, and (ii) the gain in terms of WCRT in comparison with an existing analysis for partitioned parallel tasks derived in the context of distributed system. Although most of the available results for distributed systems have been implemented in the MAST tool chain [23], only the holistic analysis originally developed by Tindell and Clark [5] and refined by Palencia et al. [6] has support for multi-path constructs in the form of fork-join tasks (not general DAGs). Thus, we restrict our attention to task sets comprised of $n-1$ sequential tasks and 1 fork-join task, where the fork-join task is the task under analysis.

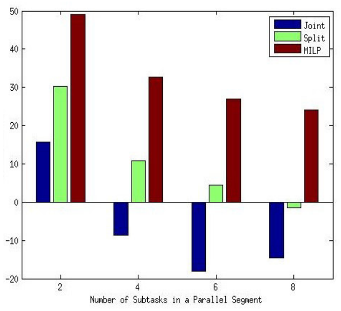
All the task sets were generated using the `randfixedsum` algorithm [24], allowing us to choose a constant total task set utilization for a given number of tasks and bounded per-task utilization. The total utilization was set to 50% of the platform capacity. For the sequential tasks, the per-task utilization ranged from $[0.05, 0.70]$, while periods were uniformly distributed over $[100, 1000]$. The task execution requirements were calculated from the respective periods and utilizations. For the fork-join task, it is workload was set to half of the maximum period (i.e., 500), whereas the WCET of each subtask was uniformly distributed over $[1, 100]$. By default, the fork-join task had 2 parallel segments with 4 subtasks within each segment. All the mapping decisions were completely random. For this reason, we study the computed WCRT and not the schedulability of the task. Thus the period of the fork-join task was arbitrarily large. We generated 100 task sets per combination of parameters, while ensuring that all the sequential tasks were always schedulable.

For the first set of experiments, we fixed the number of cores to $m = 4$ and the number of tasks to $n = 12$, while varying the number of parallel segments from 1 to 4 and the number of subtasks within a parallel segment from 2 to 8. Fig. 4a–b show the average gain (w.r.t. the WCRT) attained by our analysis comparatively to the holistic analysis, when using the three different tests for self-suspending tasks. These tests are referred to as Joint, Split and MILP, respectively to the order they were presented in Section V-E. MILP is the only test that outperforms entirely the holistic analysis, with average gains within 20 to 50%. Interestingly, Split exhibits a considerable gain when the number of parallel segments is minimized but ends up in deficit. This behavior can be justified by a higher number of self-interfering subtasks, since they are accounted once in each execution region. Joint has a drastic performance degradation when the varying parameters are increased because the number of self-suspending tasks observed in the paths grows significantly.

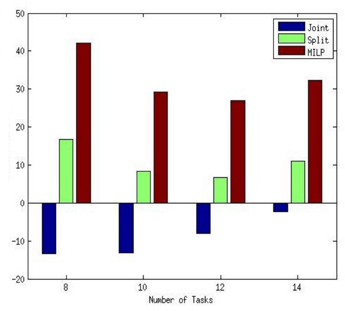
We then study the importance of light and heavy tasks to the inter-task interference. Hence, the second set of experiments had the number of parallel segments fixed to 2 and the number of subtasks in a parallel segment fixed to 4, while we varied the number of tasks in the range $[5, 20]$ and the number of cores in the range $[4, 10]$. The results are depicted in Fig. 4c–d. Both MILP and Split outperform the holistic analysis with average gains close to 30% and 10%, respectively. The holistic



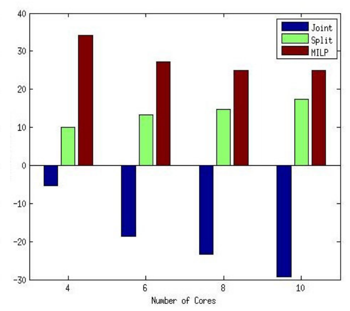
(a) $n = 12$ and $m = 4$.



(b) $n = 12$ and $m = 4$.



(c) 2 parallel segments with 4 subtasks each.



(d) 2 parallel segments with 4 subtasks each.

Fig. 4: Average WCRT gain (a)–(d) found by our approach under various system config.

analysis was derived for arbitrary deadline systems, thus assumes that some subtasks may interfere with themselves. An additional source of pessimism is the individual worst-case scenarios assumed for the subtasks. As the utilization of the interfering tasks increase, Split becomes more competitive mainly because the upper-bounds on suspension time tend to also increase. Inversely, Joint performs very poorly with average losses within 2 to 30% as more workload is assumed to interfere with the suspensions. Although not reported here, Joint becomes a reasonable alternative solution to the MILP when the ratio between the workload of the parallel task and the periods of the interfering tasks is smaller.

VIII. Conclusions

Although parallel tasks have received recently considerable attention from the real-time community, most of the available results focus on multiprocessor global scheduling. Instead, in this paper, we studied parallel tasks under partitioned scheduling as a way to minimize the negative effects of such highly parallel models on the lower-level context-dependent timing analysis. We proposed a response time analysis for sporadic DAG tasks to be fixed-priority scheduled on a multicore platform in a partitioned fashion. As the analysis is based on the self-suspending tasks theory, we derived a method to model and characterize the worst-case scheduling scenario of a partitioned DAG task as a set of self-suspending tasks. Furthermore, we showed how to transform existing response time analysis for sporadic self-suspending tasks in uniprocessors to analyze partitioned DAG tasks; both simple and more complex techniques. Experiments among randomly generated task sets show that our approach obtains a significant gain in terms of computed WCRT comparatively to the state-of-the-art. As future work, we will consider the problem of how to map parallel tasks to cores in order to improve schedulability.

Acknowledgments

This work was partially supported by National Funds through FCT/MEC (Portuguese Foundation for Science and Technology) and co-financed by ERDF (European Regional Development Fund) under the PT2020 Partnership, within the CISTER Research Unit (CEC/04234); also by the European Union under the Seventh Framework Programme (FP7/2007-2013), grant agreement nr. 611016 (P-SOCRATES), and the EU ARTEMIS JU within project ARTEMIS/0003/2012 - JU grant nr. 333053 (CONCERTO).

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