Markov Chain Modelling of Probabilistic Real-Time Systems

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Real-Time System and Safety

• Safety: ensure the correct working of R/T system; functional and non-functional aspects verified

• Modelling perspective: Ensure Task Deadlines are met

• R/T analysis required to verify this, in turn to ensure safety
Why Probabilities

• Task execution time: WCET, worst case
• Not all instances are worst case
• Over pessimistic analysis
• Each execution time \((0, WCET]\) has a probability of occurrence
• Representation of many possible execution times by a random variable
• Probability can quantify the pessimism
Probabilistic Real-Time System (pRTS)

- R/T system with at least one probabilistic parameter
- WCET replaced by probabilistic WCET (pWCET)
- Each task has pWCET which is an upper bound probabilistic distribution of all possible execution times
- pWCET gives the probability that an execution $\in (0, WCET]$ occurs
Objective

• To perform probabilistic Schedulability analysis of pRTS
• Task set is given with one or more probabilistic parameters (pWCET, Deadline, Period)
• Model stochastic occurrences and interactions (execution, delays, preemptions, etc.)
• Provide probabilistic guarantees
• Probabilistically answer the question of safety

Formal methods for pRTS

• Formal methods: mathematically proven techniques for specification and verification of a system
• Reliable: safe and accurate because of mathematical proofs available
• Model checking can be readily performed
• Model non-determinism and probability
• Handling continuous distribution (exponential, erlang, etc.)
• Feasible complexity
A model approach

• Markov Chain (MC): set of states and transitions, memoryless property

• Continuous Time MC: transitions labelled with rates, duration of transition is exponentially distributed (choice and probability)

• Other stochastic formal methods
  • Stochastic Petri Net: Problem too complex
  • Stochastic Timed Automata: inability to model probabilistic duration of execution
Current Status

- Task set at input, tasks can be probabilistic and/or deterministic; execution time, period and deadline
- Probabilistic task have pWCET for execution time
- Scheduling policy assigns each job a priority (EDF, FP)
- Number of preemptions are known, since dealing with distributions with base \((0, \infty)\)
For $\tau_2$:

- Execution times and delays encoded in transition rates
- Deadlines checked as property

$\Pr(\text{time} = 3; \text{state} = P_2) = \Pr(\text{DM})$
Limitations

• CTMC transitions are exponentially distributed
• Rigid time constraints like killing of job at deadline is not possible yet
• Convolutions are too heavy
• Hyperperiod delays not accounted for
Open Problems and Future Work

• Current methods is in the process of publishing
• Next steps: Imposing rigid timing constraints
• Avoid convolution by different interpretation of the requirement
• Integration with random variables as limits
• Hybrid stochastic model (Automata+ CTMC; CTMC+DTMC, CTMC+CTMC): to impose hard timing constraints (preemption, killing job, etc.)
Thank you
Questions